Towards a Unified Approach for Stretch Correction in Prestack Migration

Y. Liu[1], Y. Luo[2]

[1] Aramco Beijing Research Center, Aramco Asia; [2] EXPEC Advanced Research Center, Saudi Aramco

Summary

Wavelet stretch occurs in prestack imaged data regardless of migration algorithms. We propose to correct the stretch effects with a unified approach that can be adapted to different migration algorithms. We demonstrate that this unified method can be regarded as least-squares migration based on time-shift imaging condition. We then apply this method to NMO stack to verify its capability in removing NMO stretch. Numerical tests on synthetic data validate that the proposed method can provide high-resolution stacked image that is comparable to the zero-offset profile.

Introduction

Wavelet stretch is present in prestack imaged data regardless of the migration algorithm (Levin, 1998). The presence of wavelet stretch not only degrades stack resolution, it also increases uncertainty in velocity analysis and alters AVO/AVA behaviours at large angles. There have been many attempts to solve the stretch problem. The most universal approach is front-end muting the samples that have suffered severe stretch. However, muting may leave very little fold at early times, reducing the noise suppression provided by stacking. Second, muting to remove stretch is most severe at far offsets, reducing the multiple suppression provided by stacking, decreasing sensitivity to velocity errors, and losing far-offset information for AVO analysis.

Lots of efforts have been focused on alleviating NMO stretch (Dunkin and Levin, 1973; Barnes, 1992; Trickett, 2003). Rupert and Chun (1975) proposed perhaps the first non-muting solution for NMO stretch, which they called block move sum NMO. This method subdivides traces into overlapping blocks of samples, and then applies constant-shift NMO to each block. After that, these blocks are weighted and summed to form the NMO-corrected gather. In this abstract, we extend the idea of block move sum NMO to a unified approach that can correct wavelet stretching in prestack migration. We start by reviewing the wavelet stretch issue in traditional seismic imaging. We then illustrate that the stretching problem can be solved by least-squares migration based on time-shift imaging condition. Next, we use NMO stack as the migration operator to validate the proposed method. Finally, we demonstrate that the proposed method improves the image quality and lateral resolution using synthetic data tests.

Theory

A traditional imaging condition for prestack migration consists of time cross-correlation at every image location between the source and receiver wavefields, followed by image extraction at zero time (Claerbout, 1985). This imaging condition can be expressed in the Fourier domain as

$$I(\mathbf{x}) = \int\int U_s^*(\mathbf{x}, \mathbf{x}_s, \omega) U_r(\mathbf{x}, \mathbf{x}_s, \omega) d\mathbf{x}_s d\omega, \tag{1}$$

where $\mathbf{x}$ is the image location, $\mathbf{x}_s$ is the source location, ω is the angular frequency, U_s and U_r are source and receiver wavefields that can be reconstructed by

$$U_s(\mathbf{x}, \mathbf{x}_s, \omega) = G(\mathbf{x}, \mathbf{x}_s, \omega) W(\mathbf{x}_s, \omega), \tag{2}$$

$$U_r(\mathbf{x}, \mathbf{x}_s, \omega) = \int G^*(\mathbf{x}, \mathbf{x}_r, \omega) D(\mathbf{x}_r, \mathbf{x}_s, \omega) d\mathbf{x}_r. \tag{3}$$

In equations (2) and (3), $W(\mathbf{x}_s, \omega)$ is the source wavelet at $\mathbf{x}_s$, $D(\mathbf{x}_r, \mathbf{x}_s, \omega)$ is the seismic data acquired at $\mathbf{x}_r$ due to a source at $\mathbf{x}_s$, and $G(\mathbf{x}, \mathbf{y}, \omega)$ is the far-field Green's function that represents seismic response at $\mathbf{x}$ stimulated by an impulse at $\mathbf{y}$. After inserting equations (2) and (3) into (1), we can get

$$I(\mathbf{x}) = \int\int\int W^*(\mathbf{x}_s, \omega) G(\mathbf{x}, \mathbf{x}_s, \omega) G^*(\mathbf{x}, \mathbf{x}_r, \omega) D(\mathbf{x}_r, \mathbf{x}_s, \omega) d\mathbf{x}_r d\mathbf{x}_s d\omega. \tag{4}$$

In practice, the source wavelet $W(\mathbf{x}_s, \omega)$ is often ignored during prestack migration. The two Green's functions in equation (4) constitute a migration operator $K(\mathbf{x}, \mathbf{x}_r, \mathbf{x}_s, \omega)$ that can be further approximated under the high-frequency assumption as

$$K(\mathbf{x}, \mathbf{x}_r, \mathbf{x}_s, \omega) \approx e^{-i\omega t(\mathbf{x}, \mathbf{x}_r, \mathbf{x}_s)}, \tag{5}$$

where $t(\mathbf{x}, \mathbf{x}_r, \mathbf{x}_s)$ is the traveltime needed to propagate from the source at $\mathbf{x}_s$ to the subsurface scatterer at $\mathbf{x}$ and then scatter to the receiver at $\mathbf{x}_r$. The traveltime $t(\mathbf{x}, \mathbf{x}_r, \mathbf{x}_s)$ can be calculated by ray tracing or by solving Eikonal equation. With the approximation (5), equation (4) simplifies to

$$I(\mathbf{x}) = \int\int\int e^{-i\omega t(\mathbf{x}, \mathbf{x}_r, \mathbf{x}_s)} D(\mathbf{x}_r, \mathbf{x}_s, \omega) d\mathbf{x}_r d\mathbf{x}_s d\omega. \tag{6}$$

From equation (6), we can observe that traditional migration assumes that every point in the earth's subsurface is a potential scatterer, generating an idealized broadband impulse response. However, seismic data are band-limited. When applying this point-by-point migration operator (5) to the band-limited seismic data, it spreads the data out in image space, giving rise to wavelet stretch at the farther offsets. These distortions are due to the nonparallelism of the local traveltime of each reflection event (Trickett, 2003).

To prevent the stretching problem, a constant moveout should be used at any point within a reflected wavelet. This could be implemented by the following equation:

$$I(\mathbf{x},\tau)=\iiint e^{-i\omega t(\mathbf{x},\mathbf{x}_r,\mathbf{x}_s)}e^{-i\omega\tau}D(\mathbf{x}_r,\mathbf{x}_s,\omega)d\mathbf{x}_r d\mathbf{x}_s d\omega. \tag{7}$$

where τ is the time lag within which the moveout is kept constant. The resulting $I(\mathbf{x},\tau)$ is an extended image that has an additional dimension of time shift. We should note that equation (7) is exactly the time-shift imaging condition that has been well studied (for example, Faye and Jeannot, 1986; MacKay and Abma, 1992; Sava and Fomel, 2006). However, little attention has been paid to its capability of stretch correction in prestack migration.

There are mainly two challenges when applying this approach to correct the wavelet stretch in prestack migration. Since the wavelet duration is often nonstationary, the first challenge is how to set the length of the time lag. Our strategy is first setting the time window long enough to cover the longest wavelet in the data, and then optimizing the window size automatically by matching the observed data with the synthetic data simulated by the extended image. The objective function for inversion is a traditional one:

$$J=\iiint[d_{cal}(\mathbf{x}_r,\mathbf{x}_s,t)-d_{obs}(\mathbf{x}_r,\mathbf{x}_s,t)]^2 d\mathbf{x}_r d\mathbf{x}_s dt\ , \tag{8}$$

where d_{obs} is the observed data, and d_{cal} is the synthetic data simulated by the following forward operator expressed in the Fourier domain:

$$D_{cal}(\mathbf{x}_r,\mathbf{x}_s,\omega)=\iiint e^{i\omega t(\mathbf{x},\mathbf{x}_r,\mathbf{x}_s)}e^{i\omega\tau}I(\mathbf{x},\tau)d\boldsymbol{x}d\tau d\omega. \tag{9}$$

Another challenge of this method is how to stack the extended image. For wave-equation implementation, Xu et al. (2014) proposed to apply a second-pass migration to correct the non-zero-lag time-shift images and then the time-shift common image gathers (CIGs) can be flattened and stacked. This second-pass migration is similar to zero-offset depth migration and could be adapted to ray-based methods. In flat-layered media, prestack migration is essentially an NMO stack. In this simplest case, the method implementing equation (7) is called time-shift NMO (TS-NMO), and the method implementing least-squares migration based on the time-shift imaging condition is called least-squares time-shift NMO (LS-TS-NMO).

Examples

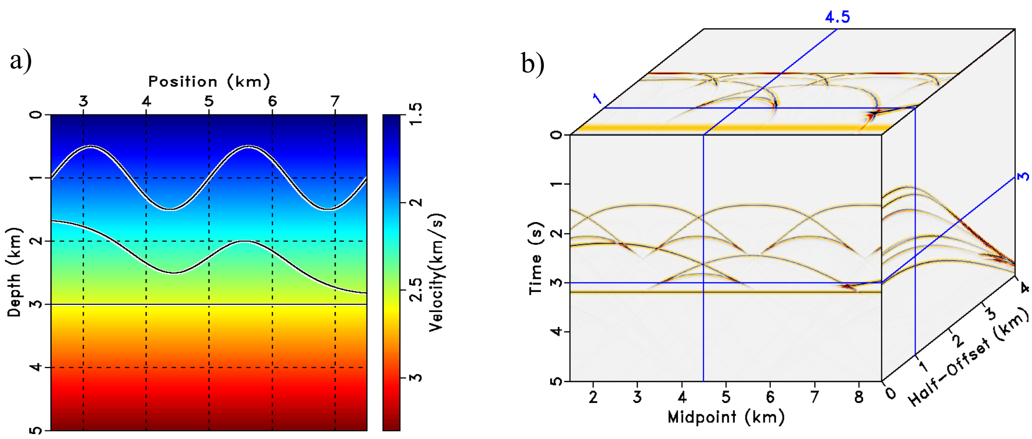

Figure 1 *(a) Velocity model overlayed by three reflectors. (b) Synthetic prestack data.*

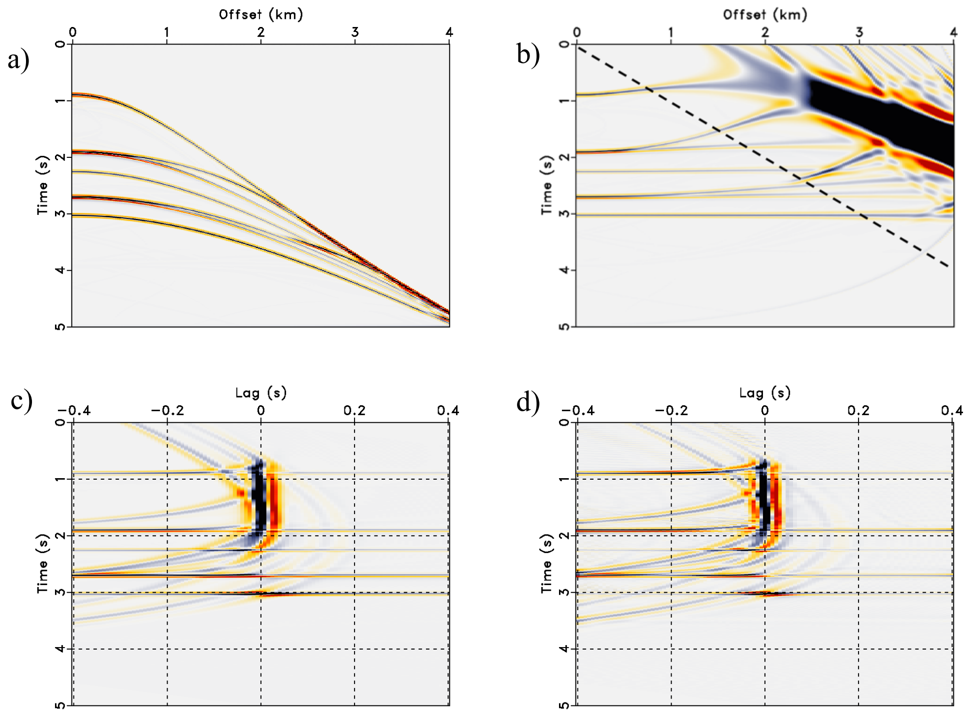

Figure 2 *(a) The input CMP gather. (b) NMO-corrected CMP gather. (c) Time-shift CIG by TS-NMO. (d) Time-shift CIG by LS-TS-NMO.*

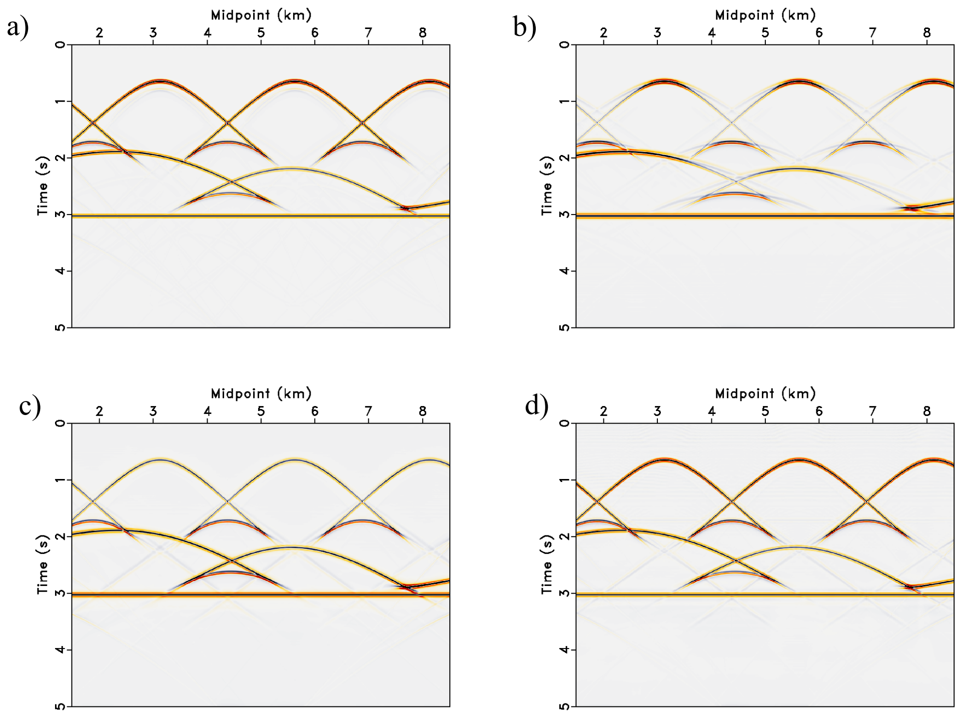

Figure 3 *(a) Zero-offset profile. (b) Stacked section after conventional NMO and manual muting such as that of Figure 2b. (c) Stacked section after TS-NMO. (d) Stacked section after LS-TS-NMO.*

In this section, we use NMO stack as the prestack migration operator to illustrate the effectiveness of the proposed stretch correction approach. Figure 1a shows a constant-gradient velocity model overlayed by three reflectors. Figure 1b shows the prestack data simulated by the Kirchhoff method. The maximal half-offset is 4 km.

Figure 2a shows a CMP gather at the position of 5 km in the velocity model. After NMO correction, the CMP exhibits severe stretching effects particularly at the far offsets, as shown in Figure 2b. The application of the front-end mute would cause the loss of many parts of the corrected gather, especially for the first two events (see the mute function indicated by the black dash line in Figure 2b). Figures 2c and 2d show the time-shift CIGs after TS-NMO and LS-TS-NMO, respectively. Comparing these two CIGs show that the image resolution has been enhanced after least-squares inversion. In addition, the time-shift CIGs have been flattened and can be stacked afterwards to provide stretch-free images.

Figure 3a shows the zero-offset profile. Figure 3b shows the stacked section after conventional NMO and manual muting such as that shown in Figure 2b. Figures 3c and 3d show the stacked section after TS-NMO and LS-TS-NMO, respectively. These results demonstrate that the proposed unified approach corrects NMO stretching successfully and provide high-resolution stacked image that is comparable to the zero-offset profile.

Conclusions

We proposed a unified approach to correct wavelet stretch in seismic migration. Although we validate the method using NMO stack in the numerical tests, this method can be easily extended to different types of prestack migration algorithms. This is the first study, to our knowledge, that provides a unified approach to solve the long-standing wavelet-stretch problem in seismic data processing.

Acknowledgements

We would like to thank Saudi Aramco for the permission to publish this work.

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