3-D seismic data regularization based on iteratively re-weighted least-squares inversion

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Summary Regularization of irregular spaced seismic data is a crucial step of data processing. In this paper, a data regularization method based on iteratively re-weighted least-squares inversion is proposed. The method can eliminate the effect of bursty noise on the interpolated data. The weighting operator is introduced to weight the data fit residual under Cauchy norm. Local plane wave model constraint is used as prior information to make the process of inversion stable and interpolate the aliased data correctly. The inverse problem is solved by the preconditioning conjugate gradient method with fast convergence. Parallel processing along time slices for three dimensional data cube can promote the efficiency of three dimensional data regularization further. Experimental results on theoretical model and real seismic data show that the proposed method is fast, efficient and applicable.

Keywords Three dimensional data regularization; B-spline; local plane wave model; preconditioning; iterative re-weighted least-squares inversion

1 Introduction

As the rapid development of three-dimensional seismic exploration, Acquiring regularly positioned data has become very difficult. While most of conventional three-dimensional seismic processing methods are specified to the regularly sampled data, the irregularity of acquired data will degrade the quality of seismic data processing, especially Kirchhoff pre-stack migration [5], multiple elimination [6] [7], Common-azimuth migration[8][9] [10]and 4-D seismic monitoring [11][12].

Many data regularization methods have been proposed recently. One group of methods is based on different types of integral (Kirchhoff) continuation operators, which can be applied directly for missing data regularization[13][15]. However, they do not behave well for continuation at small distances because of limited integration apertures and they are not well-suited for interpolating neighboring records. Additionally, they suffer from irregularities in the input geometry like all integral (Kirchhoff-type) operators. The latter problems can be solved by accurate but expensive inversion to common offset (Chemingui, 1999). Another group of methods is based on non-uniform discrete Fourier transforms or Radon transforms. This group of methods has some attractive computational properties but has serious limitation to interpolate noisy and aliased data [17][18]. The third group of methods formulates data regularization as an iterative optimization problem with a convolution operator[1][2]. Convolution with prediction-error filters is a popular choice for interpolating locally plane seismic events [19]. The method has a comparatively high efficiency, which degrades in the case of large data gaps. Handing non-stationary events presents an additional difficulty. Non-stationary prediction-error filtering leads to an accurate but relative expensive method with many adjustable parameters [21][22.

Fomel (2001, 2002) followed Claerbout's iterative optimization framework and extended it to a general approach to iterative data regularization using B-spline forward interpolation and local plane-wave-destructor filters for regularizing seismic images, which can take nonstationarity into account with fewer adjustable parameters and can interpolate aliased data appropriately. In this paper, we proposed a more robust data regularization method based on iteratively re-weighted least-squares inversion. Compared with Fomel's method, it can deal with noisy data better.

2 Theory

Seismic data regularization can be regarded as an optimization problem which aims at minimizing the energy of error between the observed data **d** and interpolated data **m** under some prior information. The inverse problem can be described as:

$$\mathbf{Lm} - \mathbf{d} \approx 0 \tag{1}$$

$$\mu \mathbf{Dm} \approx 0 \tag{2}$$

Where **L** denotes forward interpolation operator; **D** denotes regularization operator; equation (1) is the data fit residual; equation (2) is the model residual; μ denotes the trade-off parameter. In the L2-norm sense, minimizing the following objective function:

$$J = \|\mathbf{Lm} - \mathbf{d}\|_2^2 + \mu \|\mathbf{Dm}\|_2^2 \tag{3}$$

Where $\|*\|_2^2$ denotes L2-norm, the inverse problem can be solved by conjugate gradient algorithm.

In order to eliminate the effect of bursty noise on the interpolated data, minimizing the energy of data fit residual under Cauchy-norm. The objective function is changed to:

$$J = \|\mathbf{Lm} - \mathbf{d}\|_{cauchy}^2 + \mu \|\mathbf{Dm}\|_2^2 \tag{4}$$

Where

$$\|\mathbf{r}\|_{cauchy}^2 = \sum_i \ln\left(1 + \frac{r_i^2}{\bar{r}^2}\right) \tag{5}$$

Equation (5) is equivalent to introduce a weighting operator related with model to the data fit residual (Clearbout, 2009). Then equation (1) is changed to:

$$\mathbf{W}(\mathbf{Lm} - \mathbf{d}) \approx 0 \tag{6}$$

Where **W** denotes weighting operator, which can be expressed as:

$$\mathbf{W} = diag\left(\frac{1}{\sqrt{1 + r_i^2 / \bar{r}^2}}\right) \tag{7}$$

Where $\bar{r}$ denotes adjustable parameter, we can set it to be proportional to the median or some other percentile of residual vector **r**.

In order to accelerate the convergence rate of iteration, the preconditioning operator **P** (**P**=$\mathbf{D}^{-1}$) is introduced and a new vector **p** is defined, satisfying the relation:

$$\mathbf{m} = \mathbf{Pp} \tag{8}$$

The optimization problem becomes to:

$$\mathbf{W}(\mathbf{LPp} - \mathbf{d}) \approx 0 \tag{9}$$

$$\mu\mathbf{p} \approx 0 \tag{10}$$

The vector $\mathbf{p}$ can be solved by conjugate-gradient algorithm as well, and the regularly positioned data $\mathbf{m}$ can be obtained by substitution $\mathbf{p}$ to equation (8).

The crucial aspect of data regularization is how to choose the appropriate forward interpolation operator $\mathbf{L}$ and regularization operator $\mathbf{D}$ (or corresponding preconditioning operator $\mathbf{P}$). In this paper, we use B-spline interpolation operator as forward modeling operator $\mathbf{L}$, which exhibits superior performance for any given order of accuracy in comparison with other methods of similar efficiency (Thenenaz, 2000). Moreover, we use plane-wave-destructor (PWD) as regularization operator $\mathbf{D}$, which exploits the connection between the time axis (often full-sampled) and the space axis (often under-sampled) with the help of local dip spectrum estimated by PWD in order to interpolate the aliased seismic data. The regularization operator $\mathbf{D}$ can be transformed to a minimum-phase filter with a spectral factorization algorithm, and then we can implement the preconditioning operator $\mathbf{P}$ with high-efficiency recursive deconvolution in the helical coordinate. More detail about spectral factorization and helical coordinate are described by Claerbout (2004, 2009) and Fomel (2001).

After choosing the appropriate forward operator $\mathbf{L}$ and regularization operator $\mathbf{D}$, we can solve for $\mathbf{m}$ iteratively based on Least-squares (LS) or Iterative Re-weighted Least-Squares (IRLS) inversion. In the next section, we will test the effectiveness of IRLS inverse interpolation to the noisy data using synthetic models and real data.

3 Numerical tests

3.1 1-D signal test

In the first example, the input data were random sub-sampled (with decreasing density) from a sinusoid containing some random distributed bursty noise. Figure 1(a) is the sinusoid signal with 200 samples; Figure 1(b) is the input signal with 81 samples. The forward operator $\mathbf{L}$ in this case is linear interpolation. The target is to seek a regularly sampled model on 200 grid points that could predict the data with a forward linear interpolation. Sparse irregular distribution of the input data makes the regularization enforcement a necessity. The regularization operator $\mathbf{D}$ is the 3-point prediction error filter (Claerbout, 2004). Figure 1(c) is the interpolation results based on LS inversion after 5, 10, 20, 50, 500 iterations, while Figure 1(d) is the interpolation results based on IRLS inversion after 5, 10, 20, 50, 500 iterations. Compared with these two interpolation results, we can easily see that the interpolation results based on LS inversion are ruined by the bursty noise severely, but the method based on IRLS inversion can restore the missing data appropriately after 50 iterations.

3.2 3-D synthetic data test

Figure 2(a) shows Claerbout's "qdome" synthetic model (Claerbout, 1993, 1999), which models a seismic image of a complicated sedimentary geology. In the data regularization experiment, we randomly remove 50% of the original model which have been added some random distributed bursty noise, arriving at the missing data model shown in the Figure 2(b). The local slope field we used is estimated from the original model. The forward operator is cubic B-spline interpolation and the regularization operator is local plane-wave-destructor. Figure

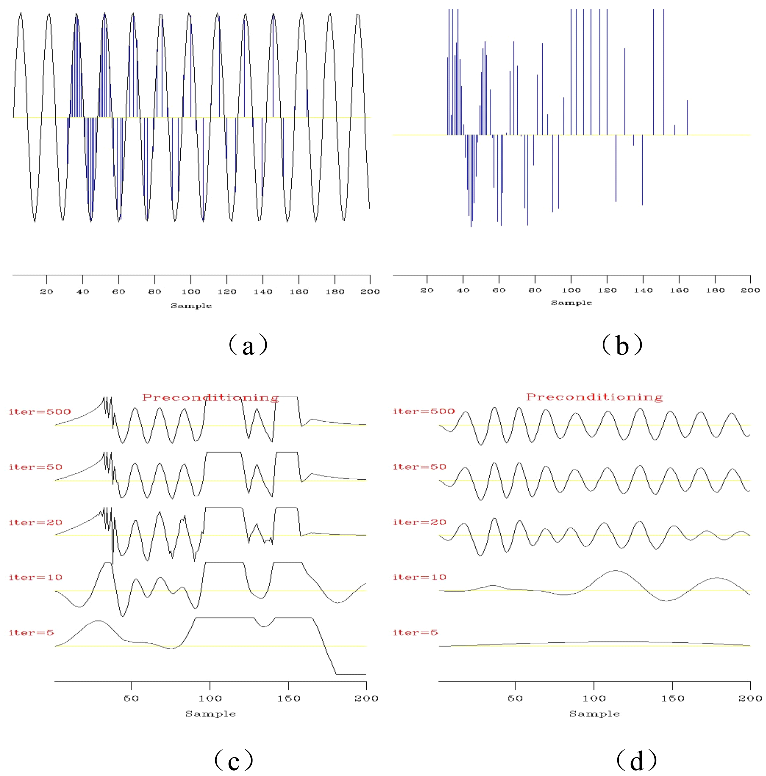


Figure 1. 1-D signal test

(a) original sinusoid signal; (b) input signal with bursty noise; (c) Interpolation results based on LS iterative inversion; (d) interpolation results based on IRLS iterative inversion

2(c) shows the interpolation result based on LS inversion, which illustrates that LS method is severely affected by bursty noise. Figure 2(d) shows the interpolation result based on IRLS inversion. Compared with original model, we can see that the IRLS method can restore most of the original signal reasonably even thought the input data contains bursty noise.

3.3 3-D real data example

A selected common offset gather in the 3-D data from Shengli oil filed is used to test our three dimensional data regularization method. Figure 3(a) shows the midpoint geometry and Figure 3(b) shows the bin fold map. Both of them illustrate the irregularity of the 3-D data. The data cube after normalized binning is shown in Figure 4(a). Binning works reasonably well in the areas of large fold but fails to fill the zero fold gaps and has an overall limited accuracy. The forward operator is cubic B-spline interpolation and the regularization operator is local PWD filter like the 3-D synthetic model test. Figure 4(b) shows the local slope field estimated by the 3-D PWD proposed by Fomel (2001). Figure 5(a) shows the interpolation result based on LS inversion and Figure 6(b) shows the interpolation result based on IRLS inversion. We can see that both of these two methods can restore the missing data appropriately, and can not easily distinguish the original and interpolated data. Figure 6(a), Figure 6(b), Figure 6(c) is the close-up of white boxes in Figure 4(a), Figure 5(a), Figure 5(b). Compared with them, we can obviously see that the IRLS method can provide more continuous events and improve the signal to noise ratio of seismic data.

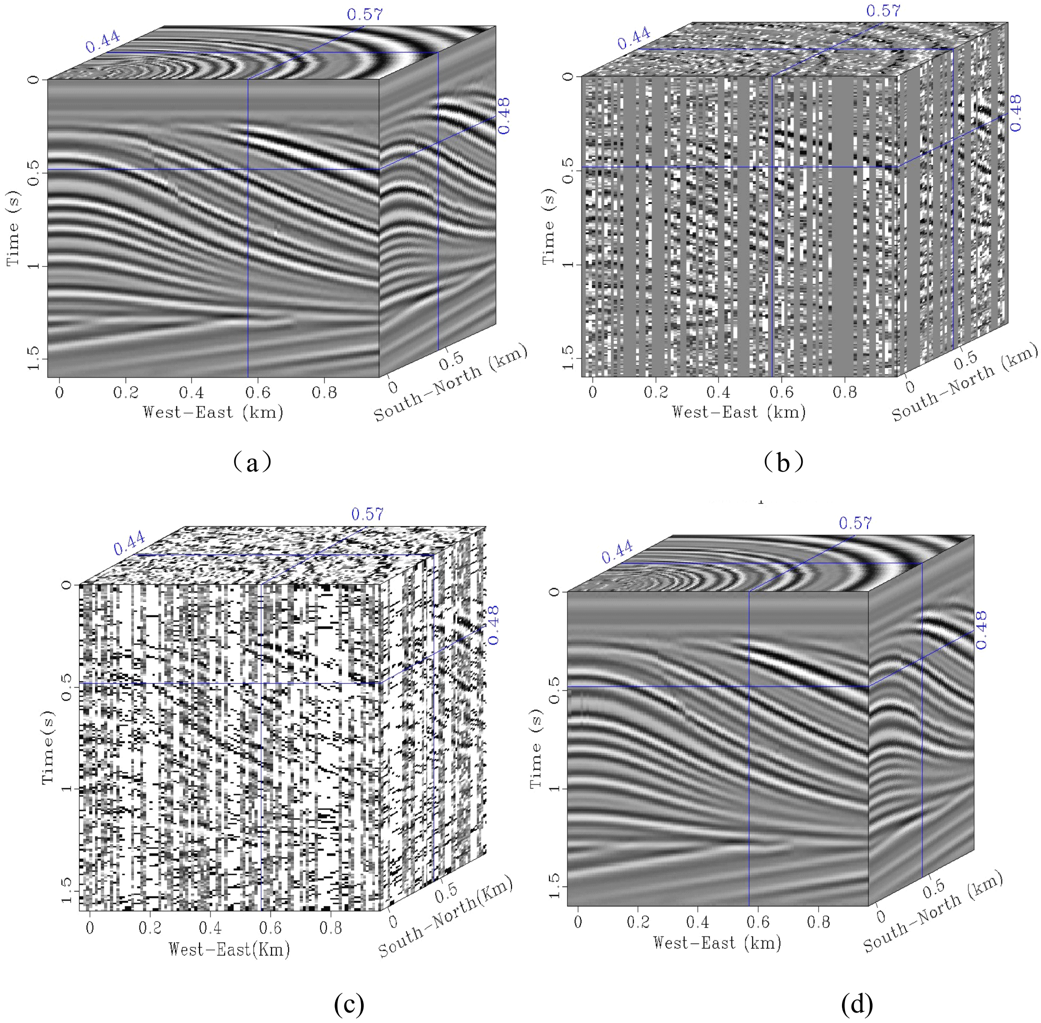


Figure 2. 3-D synthetic model test

(a) original data cube; (b) input data cube; (c) interpolation result based on LS inversion; (d) interpolation result based on IRLS inversion

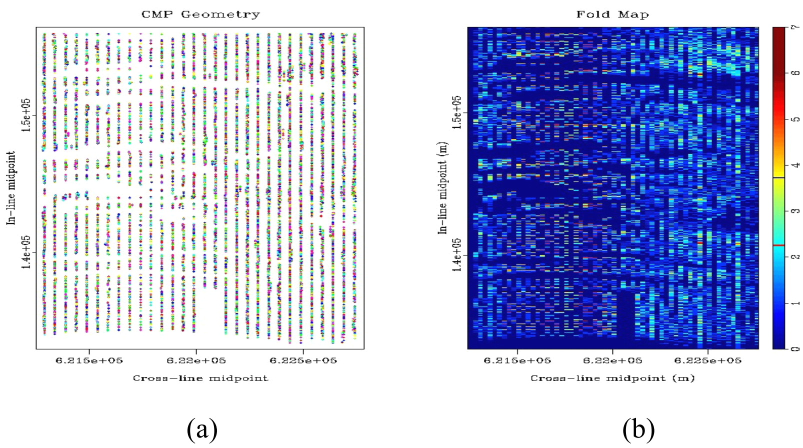
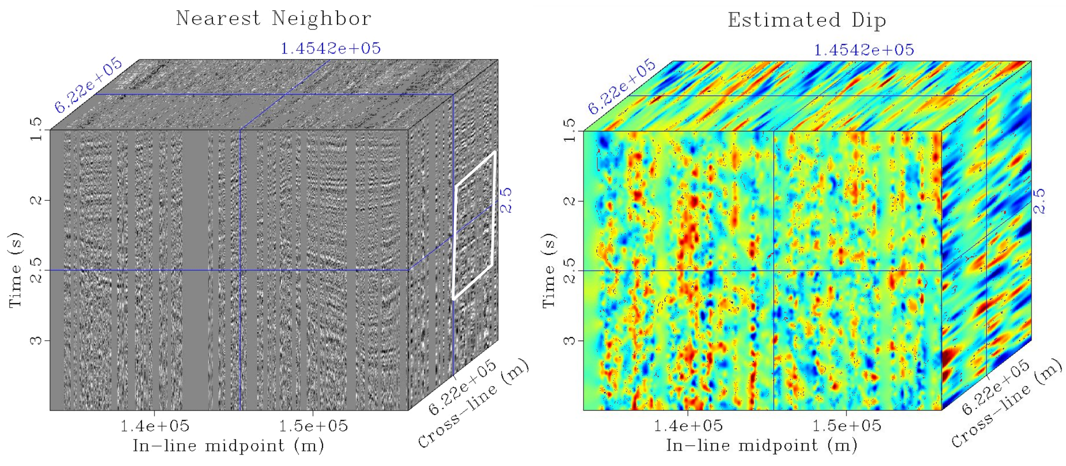


Figure 3. Geometric distribution of input data

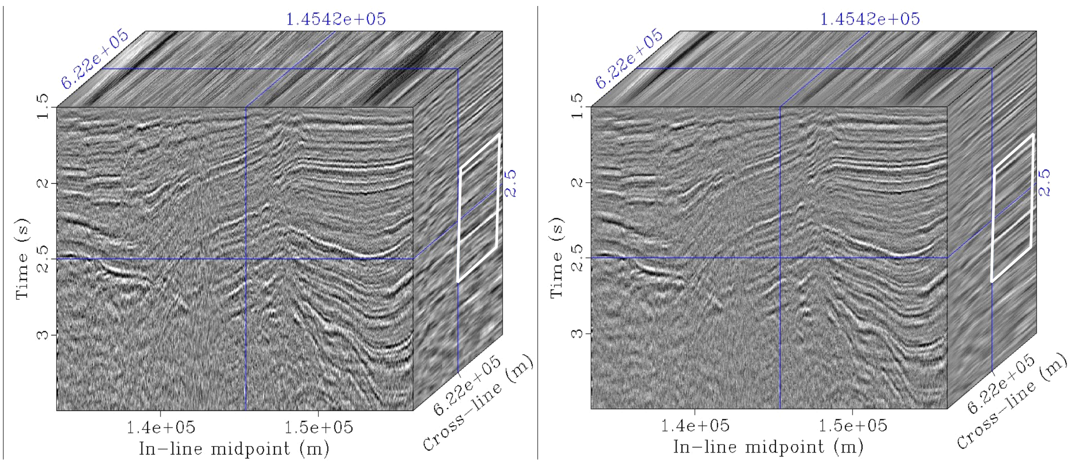
(a) Midpoint distribution for a 25 by 25 m offset bin in the Shengli oilfield dataset; (b)map of the fold distribution for the 3-D data test.



(a) (b)

Figure 4. Input data cube

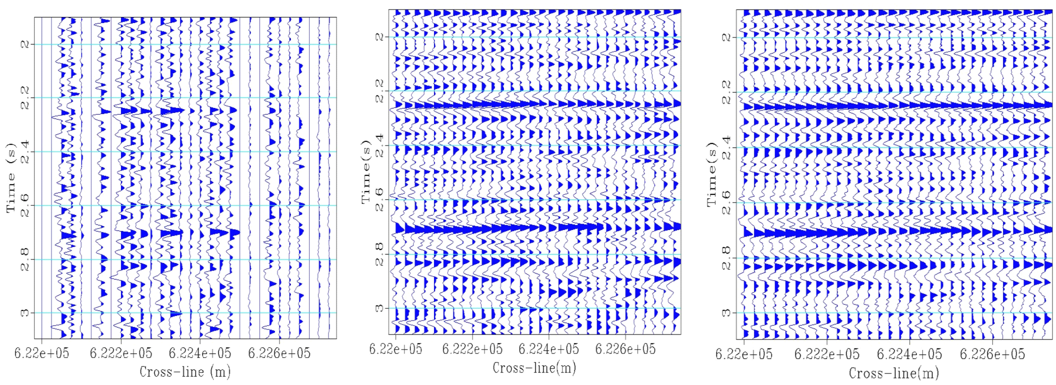
(a) 3-D data after normalized binning; (b) local dip filed estimated from (a) using 3-D PWD



(a) (b)

Figure 5. Interpolation results

(a) based on LS inversion; (b) based on IRLS inversion



(a) (b) (c)

Figure 6. Close-up comparison of white boxes in

(a) input data; (b) interpolation result based on LS inversion; (c) interpolation results based on IRLS inversion

4 Conclusions

The existence of bursty noise in the seismic data affects the convergence rate and interpolation result of seismic data regularization In this paper, compared with conventional seismic data regularization based on L2 norm inversion, we propose a new method to estimate the energy of data fit residual in the sense of Cauchy norm, equivalent to introduce a weighting function related to model, to eliminate the effects of bursty noise to interpolation result, making interpolated seismic events more continuous with higher ratio of signal to noise. Local plane wave model constraint is used as prior information to make the process of inversion stable and interpolate the aliased data correctly. The inverse problem is solved by the preconditioning conjugate gradient method with fast convergence. Parallel processing along time slices for three dimensional data cube can promote the efficiency of three dimensional data regularization further. Experiments with synthetic and real data show that the proposed seismic data regularization method based on IRLS inversion can interpolate noisy data appropriately and provide a more reasonable result than LS inversion method.

[1] Claerbout, J. F. Earth Soundings Analysis: Processing Versus Inversion. *Blackwell Scientific Publications*， 2004

[2] Claerbout, J. F. Geophysical estimation by example: Geophysical soundings image construction and Multidimensional auto-regression. *Stanford Exploration Project*,2009

[3] Sergey Fomel. Three Dimension Seismic Data Regularization. *Ph.D. thesis* , Stanford University，2001

[4] Sergey Fomel. Applications of plane-wave destruction filters. *Geophysics*, 2002, 67(6): 1946~1960

[5] Gardner, G. H. F., Canning. A effects of irregular sampling on 3-D prestack migration. *Abstracts of 64th SEG Mtg*, 1994, 1553~1556

[6] Verschuur, D. J., Berkhout, A. J. Estimation of multiple scattering by iterative inversion, part II: Practical aspects and examples. *Geophysics*,1997,62(05):1596~1611

[7] van Dedem, E. J., Verschuur, D. J. 3-D surface-related multiple elimination and interpolation. *Abstracts of 68th SEG Mtg*, 1998 , 1321~1324

[8] Biondi, B., and Palacharla, G. 3-D prestack migration of common-azimuth data: *Geophysics*, 1996a, 61(6):1822~1832

[9] Biondi, B., and Palacharla, G. 3-D prestack migration of common-azimuth data: *Geophysics*, 1996b,61(6):1822~1832

[10] Biondi, B. L. *3-D Seismic Imaging*. Stanford Exploration Project, 2004

[11] Legott, R. J., Williams, R. G., Skinner, M., Co-location of 4D seismic data in the presence of navigational and timing errors. *Abstracts of 69th SEG Mtg* , 1999,1699~1702.

[12] Morice, S., Ronen, S., Canter, P., Welker, K., Clark, D. The impact of positioning differences on 4D repeatability. *Abstracts of 70th SEG Mtg*,2000, 1611~1614

[13] Fomel, S., and Bleistein, N. Amplitude preservation for offset continuation: Confirmation for Kirchhoff data.1996, *SEP*–92, 219~227

[14] Chemingui, N. Imaging irregularly sampled 3D prestacked data. *Ph.D. thesis*, Stanford University. 1999

[15] Bagaini, C., and Spagnolini, U. 2-D continuation operators and their applications. *Geophysics*, 1996, 61(6):1846~1858

[16] Biondi, B., Fomel, S., and Chemingui, N. Azimuth moveout for 3-D prestack imaging. *Geophysics*,

1998, 63(2):574~588

[17] Liu B., D. Sacchi. Minimum weighted norm interpolation of seismic records. *Geophysics*, 2004, 69(4):1560~1568

[18] D. Trad, T. Ulrych, M. Sacchi. Latest views of the sparse Radon transform. *Geophysics*, 2003, 68(1):386~399

[19] Spitz, S. Seismic trace interpolation in the f-x domain. *Geophysics*, 1991,56(6): 785~794

[20] Guo Jiuying, Zhou Xingyuan, Yu Shoupeng. Iso-interval trace interpolation in F-X domain. *Oil Geophysical Prospecting*, 1996, 31(1):28~34

[21] William Curry. Interpolation with prediction-error filters and training data. *Ph.D. thesis*, Stanford University，2008

[22] Sean Crawley. Interpolation with smoothly non-stationary prediction-error filters. *Ph.D. thesis*, Stanford University，1999

[23] Paul Sava, Sergey Fomel. Spectral factorization revisited. *Stanford Exploration Project*， 1999, SEP100, 227~235

[24] Thévenaz, P., Blu, T., Unser, M. in Handbook of Medical Image Processing, in press , 2000

[25] Unser, M., Aldroubi, A., Eden, M. B-spline signal processing: Part I – Theory. *IEEE Transactions on Signal Processing*, 1993, 41(1): 821~832